# Research on Total Cost of Oil Pipeline Laying in Urban Area Based on Linear Programming

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**Abstract:** The laying of the oil pipeline is a big project that costs lots of money. How to lay oil pipeline between oil refinery and railway is an engineering problem worthy of attention. This paper starts from studying the laying length of common and non-common pipelines between two refineries and railway, according to whether the cost of laying the shared pipeline and the non-common pipeline is the same, and whether the connection between the two refineries and the railway line are vertical or non-vertical. It is divided into four categories by establishing different cost expressions. Considering the laying of pipelines in urban areas, compensation for demolition, etc., this paper proposes a total cost model for laying oil pipelines in urban areas based on linear programming, and finds that the most cost-effective construction of pipelines is 2,827,000 yuan. Considering the actual market situation, due to the different laying costs of oil pipelines in refineries a and b, the shortest pipeline length and the total cost of laying may not be guaranteed. The planning model was optimized, and the most cost-effective cost of pipeline construction was obtained through the Lingo software. It was 2.5197 million yuan.

## 1. The fundamental assumptions of pipeline cost model

For the laying of urban oil pipelines, the distance between the refinery and the railway line, multiple refineries provide oil at the same time, the location distribution of refineries and the railway line, the cost of different laying methods, the price of laying pipelines and other factors should be taken into account, so as to obtain the most ideal laying scheme and cost.Now take two oil refineries and one railway line as the basic model, and make the following assumptions:

(1) The distance between the two oil refineries and the distance between the two oil refineries and the railway line are within the safe range (the safe distance should be more than 1,000 meters through the relevant data);

(2) The railway line between the two refineries is approximately straight, and geological impacts are not considered during modeling (that is, some places cannot be laid due to geological reasons during construction).

(3) The price of non-shared pipelines will not be more expensive than that of shared pipelines.

## 2. Pipeline cost model based on geometric relationship

### 2.1 Problem Analysis

Since the price of the non-common tubing is the same, we only need to consider the various scenarios of the distance between the two refineries to the railway line and the distance between the two refineries, and the situation where the cost of the shared pipeline is the same as or different from the cost of the non-shared pipeline when there is a shared pipeline. To solve this problem, we set up a model of pipeline cost model based on geometric relationship below.

First, based on whether the prices of shared and non-public pipelines are the same, it is divided into two cases where the unit cost of shared and non-shared pipelines is the same and the unit cost of shared and non-shared pipelines is different. The line between the two refineries is perpendicular to the railway line and the line between the two refineries is not perpendicular to the railway line.

When the line connecting the two refineries is perpendicular to the railway line, the route is obvious and is a straight line distance.

When the connection between the two oil refineries is not perpendicular to the railway line, we first select a point O in the rectangular ACDE in Figure 2 as the joint point of the oil pipes of the two plants, which is actually the starting point of the common oil pipe. Let us assume that the length of the common oil pipe is constant, so that the joint point of the oil pipe can be moved on a straight line parallel to the railway line (in the following modeling and solving process, this line is used as the x-axis, and the O point is moved on the x-axis located in the rectangular ACDE range), in order to minimize the cost with the same price of non-shared tubing, the problem is transformed into the minimum value of the sum of the distances between the two refineries (hereinafter, the minimum value of | AO | + | BO |), so you can borrow the law of reflection to find this minimum value and determine the position of point O. Because, for any given h, such a minimum value can be found, and a minimum must be found in a series of such minimum values. This minimum value and its corresponding O-point is the answer aboutthe minimum value of the sum of the required distances between the two refineries.

When the starting point of the common oil pipe is determined, the common oil pipe must be laid in a direction perpendicular to the railway line. Only in this way can the cost be minimized. When applying the law of reflection, the x-axis is used as a mirror.

**2.2 In the case that the unit cost of the shared pipeline and the non-shared pipeline is the same**

a) When the line connecting the two refineries is perpendicular to the railway line, as shown in Figure 1 (A and B positions can be interchanged):

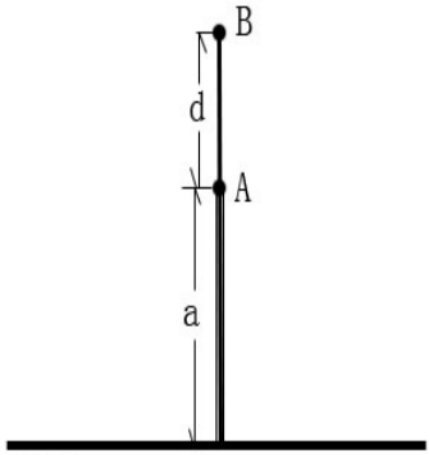


Figure 1 Structure of two refineries

$a$: Vertical distance from refinery a to railway line, $b$: The vertical distance from refinery b to the railway line.

Obviously, it costs the least to lay the non-shared pipeline from the far oil plant to the near oil plant, and then from the near oil plant to the shared station to the additional station. Therefore, the total costis $W = P(d + a)$.

b) When the line connecting the two refineries is not perpendicular to the railway line, a pipeline cost model based on a geometric triangular relationship is established, as shown in Figure 2 (where a and b are uncertain).

Since the unit cost of the two pipelines is the same, to minimize the total laying cost, the laying distance should be minimized. In order to obtain the minimum distance, the joint point o of the two pipelines must be determined.

So the problem is to find a little o in the quad abcd, so that | AO | + | BO | + | OF | is the smallest. According to the actual situation, o should be defined in the rectangular acde.

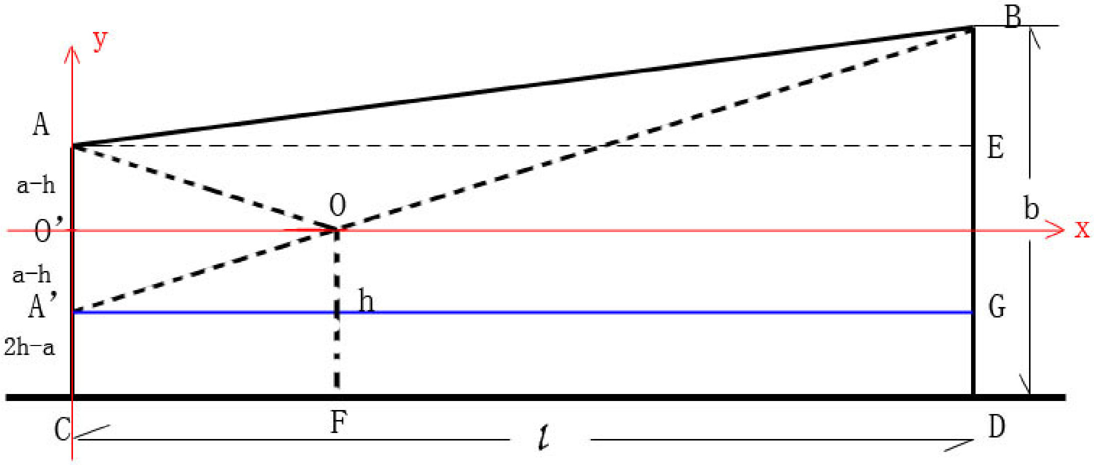


Figure 2 Pipeline cost model based on geometric relationship

$h$: Length of public pipe, $l$: The sum of the vertical distance between the two refineries and the suburban dividing line, $P$: When the prices of the non-public pipelines used by the two plants are the same, the cost per unit length of the non-public pipelines. $a \le b$ Time, $0 \le |FO| \le a$, which is $0 \le h \le a$ ;when $a > b$ Time, $0 \le |FO| \le b$ ,which is $0 \le h \le b$ .

Assume that point O is shown in Figure 2. Now the line where AC is located is taken as the y-axis, passing O and $OO' \perp AC$ to $O'$, to $O'$ as the origin, $OO'$ for the x axis, establish a rectangular coordinate system as shown in the figure2.

For any given | OF | length value h, move the O point on the x axis to determine the minimum value of | AO | + | BO | accordingly. Then slide up and down the x axis to get | AO | + | BO | the minimum value at different h values.

Applying the principle of specular reflection in physics, taking the x-axis as the axis of symmetry, making A point a mirror point A ', connecting A'B, we can know that | A'B | is the minimum value of | AO | + | BO |. Knowing:

$$|A'O'| = |AO'| = a - h \tag{1}$$

$$|A'C| = |GD| = 2h - a \tag{2}$$

$$|A'G| = |CD| = l \tag{3}$$

$$|BG| = b + a - 2h \tag{4}$$

In $Rt\Delta BA'G$ , Obtained from the Pythagorean theorem:

$$|A'B| = \sqrt{l^2 + (b + a - 2h)^2} \tag{5}$$

$$(|AO| + |OB|)_{\min} = \sqrt{l^2 + (b + a - 2h)^2} \tag{6}$$

$$(|OF| + |AO| + |OB|)_{\min} = \sqrt{l^2 + (b + a - 2h)^2} + h \tag{7}$$

In summary, the total cost of constructing the pipeline is w:

when $a \le b$ Time, $W = P\left(\sqrt{l^2 + (b + a - 2h)^2} + h\right)$ $(0 \le h \le a)$

when $a > b$ Time, $W = P\left(\sqrt{l^2 + (b + a - 2h)^2} + h\right)$ $(0 \le h \le b)$

**2.3 In the case that the unit cost of the shared pipeline and the non-shared pipeline is different**

a) When the line connecting the two refineries is perpendicular to the railway line, the scenario is the same as in Figure 1:

Obviously, the cost is the least from laying the non-shared pipelines in the far oil plant to the near oil plant, and then laying the common pipelines in the near oil plant to the additional station. At this time, the total cost is $W = Pd + P'a$

b) When the line connecting the two refineries is not perpendicular to the railway line, it is the same as in Figure 2 (where a and b are uncertain in size, it may be assumed $a \le b$ )：

Because the cost of the two types of pipeline units is different, but the cost of the non-shared pipeline units is the same. When the length of the shared pipeline is given, in order to minimize the cost, the sum of the lengths of the non-shared pipelines must be minimized. At this time, the model can still be applied. The unit cost used to calculate the total cost of the shared pipeline and the total cost of the non-shared pipeline is different.

The calculation process of total pipeline construction costs is as follows:

$$(|AO|+|OB|)_{\min} = \sqrt{l^2 + (b+a-2h)^2} \tag{7}$$

$$W = P\left(\sqrt{l^2 + (b+a-2h)^2}\right) + P'h \tag{8}$$

## 3. Model of total cost of laying oil pipes in urban areas based on linear programming

### 3.1 Problem Analysis

It is assumed that the laying cost of all pipelines is 72,000 yuan / km. When laying oil pipes in urban areas, additional costs caused by the demolition of buildings need to be paid, and the distance from a refinery to the railway line is less than b We assume that the urban oil pipeline enters the suburb from b1 of the suburban dividing line (i.e., point b1 in Figure 3 below), and the following are the results of the three engineering consultants to estimate this surcharge.

Table 1 Additional costs of the three engineering companies

| Engineering Consulting Company | Company 1 (Class A qualification) | Company II (Class B qualification) | Company III (Class B qualification) |
|---|---|---|---|
| Additional cost (ten thousand yuan / km) | 21 | 24 | 20 |

When solving suburban expenses, a <b and the connection between the two refineries is not perpendicular to the railway line.Now it is necessary to solve the problem of the cost about laying oil pipes in the urban area.This part of the cost includes the laying of oil pipes and the compensation for demolition.

Regarding the cost of tubing laying, the length of the tubing in the urban area (hereinafter referred to as bb1 |) is multiplied by its price to obtain it; and the relevant demolition compensation fees have been given as reference values though the three engineering consulting companies , We use the corresponding weight given to the three companies according to the situation, and then based on the weight and the corresponding unit length cost estimate, the expected value of the unit length cost estimate (hereinafter referred to as p0) is calculated, and then this expected value which is multiplied by the length of the oil pipe in the urban area can be obtained. Then the two costs are added to obtain the total cost of laying the oil pipe in the urban area. At this point, problem two can be solved and the planning model two can be established accordingly.

### 3.2 Total cost model of laying oil pipes in urban areas based on linear programming

Compared to the first problem, the problem of laying pipes in urban areas was added to the second problem.

Suppose the location of the pipeline laid by refinery b from the urban area to the suburbs is $B_1$, The projective point of b on the suburban dividing line is $B_2$ .

Now consider the situation of laying pipelines in urban areas, because the costs of laying pipelines in urban areas include pipeline laying costs and demolition compensation fees, and the length of pipelines laying in urban areas can be expressed by the Pythagorean theorem as $\sqrt{\left(l-c\right)^2+m^2}$, For the length of the pipe laying in the suburbs, a pipe cost model based on the geometric triangle relationship can be borrowed, as shown in the figure below.

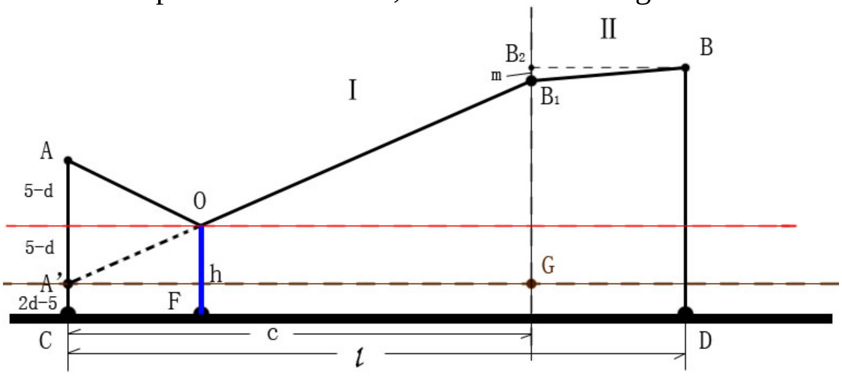


Figure 3 Model of the length of the pipe laying in the suburb based on the geometric relationship

$c$: Vertical distance from refinery a to the suburban dividing line, $d$: Distance between two refineries, $P'$: Cost of unit length of public pipeline, $p_0$: Expected value of the engineering consulting company's assessment of the additional cost of the unit length of pipeline when the pipeline is laid in the urban area, $P_1$**:** a Cost of unit length when non-public pipelines are laid in an oil refinery, $P_2$**:** b Cost of unit length when oil refinery is laying non-public pipelines, $W$: Total cost of pipeline construction. Because for any given common pipeline length h, AO, $B_1$The pipe length and the total length of the o section can be obtained by the principle of light reflection:

$$(|AO|+|B_1O|)_{\min}=\sqrt{c^2+(b-m+a-2h)^2} \tag{9}$$

Therefore, the total cost of laying the pipeline can be expressed as a function of h and m:

$$f(h,m)=P(\sqrt{c^2+(b-m+a-2h)^2}+h)+(P+P_0)(\sqrt{\left(l-c\right)^2+m^2}) \tag{10}$$

According to the intent, we can get $0\le h\le a$ , $0\le m\le b$ .among them $a=5, b=8, c=15, l=20, P=7.2$ are all known. $P_0$ is the expected value of the demolition cost per unit length. According to the estimation of the additional costs of demolition and engineering compensation by the three engineering consulting companies in the title (see Table 1 for details), the weight can be obtained as follows:

Because Company One has Grade A qualifications, and Company Two and Company Three have Grade B qualifications, the weight of Company One can be set to 50%, and the weights of Company Two and Three are both 25%.

$$P_0=21\times 50\%+24\times 25\%+20\times 25\%=21.5 \text{ (Ten thousand yuan)} \tag{11}$$

To sum up, a total cost model for laying oil pipes in urban areas based on linear programming can be established. The decision variables, objective functions, and constraints are as follows:

Decision variable: Let the length of the shared pipeline be h kilometers, and the projected length of the pipeline line laid by refinery B in the urban area on the suburban boundary line is m kilometers.

Objective function: The total cost of laying the pipeline and the additional cost of urban demolition is the least:

$$f(h,m)=7.2(\sqrt{15^2+(8-m+5-2h)^2}+h)+(7.2+21.5)(\sqrt{\left(20-15\right)^2+m^2}) \tag{12}$$

**Restrictions:**

(1) The length h of the common pipeline does not exceed the vertical distance of 5 kilometers from the A refinery to the railway, that is, $h \le 5$;

(2) The projected length m of the pipeline line laid by refinery B in the urban area on the suburban boundary line does not exceed the vertical distance of B refinery from the railway to 8 kilometers, that is, $m \le 8$;

(3) Both h and m cannot be negative, that is, $h \ge 0$, $m \ge 0$.

Solving the planning model with lingo software, we can get $h = 1.85$ (km), $m = 0.63$ (km), which can be obtained by substituting into the calculation formula (12). $f(h, m) = 282.70$ (Ten thousand yuan)

## 4. Improvement of the total cost model of laying oil pipes in urban areas based on linear programming

### 4.1 Problem Analysis

On the basis of the second question, it has been further actualized. The unit price of the shared pipeline and the non-shared pipelines of the a and b refineries are different, and other conditions are unchanged. Although the change is small, due to the unit price of non-public pipelines Change, which makes all the models above not applicable.

Because on the problem of non-shared pipelines, the shortest pipeline length and the total cost of laying may not be guaranteed, so it is no longer practical to find the shortest pipeline length and it is necessary to rebuild the model. The following model is now established to establish the rectangular coordinate system ( (See Figure 4 below for details), and set the pipeline joint point o and its coordinates, so that the coordinates of the additional vehicle station (that is, point f in Figure 4 below) are determined, and the coordinates of other points can be determined by known conditions. Calculate the length of the line segments ao, fo, b1o in Figure 4 by using the formula for calculating the distance between points in mathematics. Due to the price of the pipeline represented by the line segments ao, fo, b1o and the surcharge of the pipeline represented by b1o are all known, so the least cost of pipeline construction can be obtained.

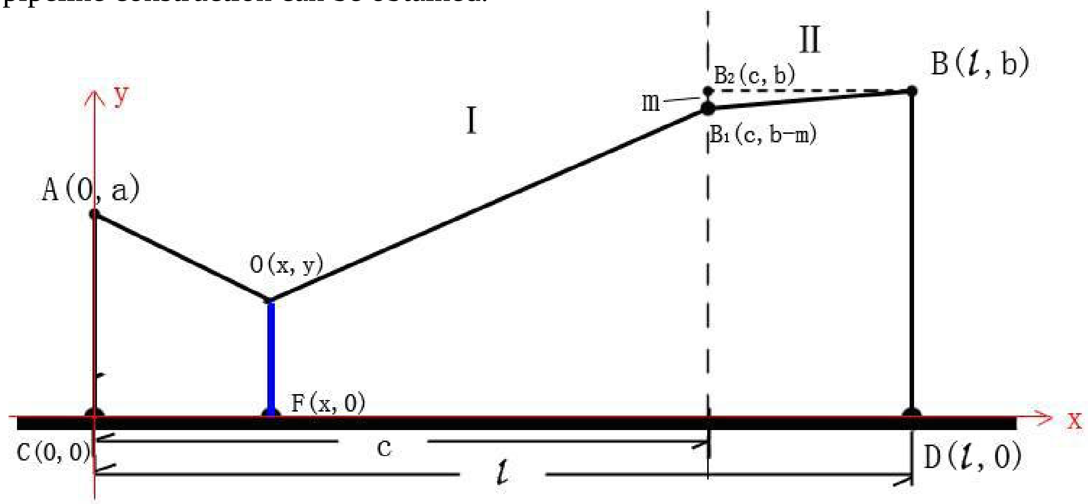


Figure 4 Model of the length of the pipe laying in the suburb based on the geometric relationship

### 4.2 Improvement of the total cost model of laying oil pipes in urban areas based on linear programming

In question three, the unit price of the shared pipeline and the non-shared pipelines of the refineries a and b are different. To further save costs, you can choose the appropriate oil pipeline according to the production capacity of the refinery. At this time, the pipeline laying costs It is 56,000 yuan / km for plant A, 60,000 yuan / km for plant B, and the shared pipeline cost is 72,000 yuan / km. Additional costs such as demolition are the same as above. Therefore, in terms of non-common pipelines, the shortest pipeline length and The total cost of laying is guaranteed to be the least. Therefore, the established model cannot be used, and the model must be rebuilt.

We take the straight line where the railway line is located as the x-axis and the straight line where the AC is located as the y-axis to establish a plane rectangular coordinate system as shown in Figure 4.

Set the node O (x, y) of three kinds of pipelines, and build a new station F (x, 0). The coordinates of other points can be indicated by the parameters shown in Figure 4. From the figure:

$$|AO| = \sqrt{x^2 + (y-a)^2} \tag{13}$$

$$|B_1O| = \sqrt{(c-x)^2 + (b-m-y)^2} \tag{14}$$

$$|BB_1| = \sqrt{(l-c)^2 + m^2} \tag{15}$$

At this time, the relationship between the total cost and x, y, m can be obtained:

$$f(x,y,m) = P_1\sqrt{x^2 + (y-a)^2} + P_2\sqrt{(c-x)^2 + (b-m-y)^2} + (P_0 + P_2)\sqrt{(l-c)^2 + m^2} + P'y \tag{16}$$

According to the intent, we can get $0 \le x \le l$ , $0 \le y \le b$ , $0 \le m \le b$ ,among them $a = 5,\quad b = 8,$ $c = 15,\quad l = 20,\quad P_0 = 21.5,\quad P_1 = 5.6,\quad P_2 = 6.0,\quad P' = 7.2$ All are known.

In summary, considering the actual situation, the total cost model of urban pipeline laying based on linear programming in Problem 2 can be practically optimized. The decision variables, objective functions, and constraints are as follows:

Decision variable: Set the distance from node O to the railway to y kilometers for three types of pipelines, and the distance from node O to refinery A and the railway vertical line for x kilometers. The pipeline line laid by refinery B in the urban area is divided in the suburbs. Projection length on the boundary is m kilometers

Objective function: The total cost of laying the pipeline and the additional cost of urban demolition is the least:

$$f(x,y,m) = 5.6\sqrt{x^2 + (y-5)^2} + 6.0\sqrt{(15-x)^2 + (8-m-y)^2} + (21.5+6.0)\sqrt{(20-15)^2 + m^2} + 7.2y \tag{17}$$

**Restrictions:**

1) The distance x from the node O of the three pipelines to the refinery A and the vertical line of the railway does not exceed the sum of the vertical distances between the two refineries and the suburban boundary line of 20 kilometers, that is, $x \le 20$

2) The distance y from node O to the railway does not exceed the vertical distance of 8 kilometers from the B refinery to the railway, that is, $y \le 8$

3) The projected length m of the pipeline line laid by Refinery B in the urban area on the suburban boundary line does not exceed the vertical distance of B refinery from the railway to 8 kilometers, that is, $m \le 8$

4) x, y, m cannot be negative, i.e. $x \ge 0$ , $y \ge 0$ , $m \ge 0$ .

Solving the planning model with lingo software, we can get $x = 6.73$ (km), $y = 0.14$ (km), $m = 0.72$ (Km). Substituting into calculation formula (17), the most cost-effective construction of the pipeline is $f(x,y,m) = 251.97$ (Ten thousand yuan)

## 5. Conclusion

In this article, the layout of oil pipelines is mainly studied from the perspective of the least construction cost. First, we will classify and discuss according to the same or different laying costs of shared pipelines and non-common pipelines. Second, due to the need to consider laying pipelines in urban areas, to the compensation for demolition, etc .. By weighting the estimated cost of the company, find the expected value. $P_0 = 21.5$ (10,000 yuan), and using the established planning model, the most cost-effective construction of the pipeline is 2,827,700 yuan, of which the length of the shared pipeline is 1.85 kilometers, and the projection of the pipeline line laid by the refinery b in the urban area to the suburban boundary line is 0.63 km. Finally, considering the actual market conditions, the laying costs of oil pipelines in refineries a and b will be different, so the shortest pipeline length and the total cost of laying may not be guaranteed, so we have established a new planning model. The lowest cost of pipeline construction obtained through the lingo software is 2.519 million yuan. The distance from the junction point o of the three pipelines to the refinery a and the railway perpendicular is 6.13 kilometers, and the distance from the junction point o to the railway is 0.14 kilometers. The projected pipeline line of refinery b in the urban area to the suburban boundary line is 0.72 kilometers.

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